The hawk at once starts in pursuit, flying at the rate of $m=5$ feet per second and keeping always in a straight line with the starting point and the hen.

Determine the path followed and the distance the hawk will fly before catching the hen.

MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo.   All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

11. Proposed by CHARLES E. MYERS, Canton, Ohio.

"A homogeneous sphere moves down a rough inclined plane, whose angle of inclination. θ to the horizon is greater than that of the angle of friction; if the coefficient of friction is less than $\frac{2}{7}$ tang. θ, show that the sphere will roll and slide down the inclined plane."

Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia·

Let $a=$ the radius of the sphere, ϕ the angle turned through by the sphere.

Resolving along the perpendicular to the inclined plane we have, if mk^2 be the moment of inertia of the sphere about a horizontal diameter,

$$m\frac{d^2x}{dt^2}=mg\sin\theta-F\ldots(1),$$ where F is friction acting along the plane at the point of contact of the sphere and mg acting vertically at the centre.

Also $m\frac{d^2y}{dt^2}=-mg\cos\theta+R\ldots(2)$, where R is the reaction perpendicular to the plane. In order to avoid reactions let us take moments about the point of contact, and we get $ma\frac{d^2x}{dt^2}+mk^2\frac{d^2\phi}{dt^2}=mga\sin\theta\ldots(3).$

Since there is no jump, $y=a\ldots(4).$

From (1) $F=\frac{2}{7}mg\sin\theta\ldots(5)$, from (2) and (4) $R=mg\cos\theta\ldots(6).$

$\therefore$ from (5) and (6) $F=\frac{2}{7}R\tan\theta$ but since $\mu=$coefficient of friction $<\frac{2}{7}\tan\theta$, $F=\mu R$ and the equations of motion become $m\frac{d^2x}{dt^2}=mg\sin\theta$

$-\mu R\ldots(7),$ $0=-mg\cos\theta+R\ldots(8).$ $ma\frac{d^2x}{dt^2}+mk^2\frac{d^2\phi}{dt^2}=mga\sin\theta\ldots(9).$

From (8) $R=mg\cos\theta$, this in (7) gives $\frac{d^2x}{dt^2}=g(\sin\theta-\mu\cos\theta),$

$\therefore$ $x=\frac{1}{2}gt^2(\sin\theta-\mu\cos\theta)$ since the sphere starts from rest.   Also $k^2=\frac{2}{5}a^2.$

$\therefore$ (9) becomes $\dfrac{d^2x}{dt^2} + \tfrac{2}{5}a\dfrac{d^2\phi}{dt^2} = g \sin\theta$. Substituting $\dfrac{d^2x}{dt^2}$ we get

$g \sin\theta - g\mu \cos\theta + \tfrac{2}{5}a\dfrac{d^2\phi}{dt^2} = g \sin\theta$.

$\therefore a\dfrac{d^2\phi}{dt^2} = \tfrac{5}{2}\mu g \cos\theta$. $\therefore \phi = \tfrac{5}{4}\mu \dfrac{g}{a} t^2 \cos\theta$. Also $\dfrac{dx}{dt}$

$$= gt(\sin\theta - \mu \cos\theta), \; a\dfrac{d\phi}{dt} = \tfrac{5}{2}\mu gt \cos\theta.$$

$\therefore \dfrac{dx}{dt} - a\dfrac{d\phi}{dt} = gt(\sin\theta - \tfrac{7}{2}\mu \cos\theta)$ is the velocity of the point of the sphere in contact with the plane. Since $\mu < \tfrac{2}{7}\tan\theta$, this velocity can never vanish. $\therefore$ the friction will never change to rolling friction. This completely determines the motion.

II. **Solution by WILLIAM HOOVER, A. M., Ph. D., Athens, Ohio.**

Let $a=$ the radius of the sphere, $k^2 = \tfrac{2}{5}a^2 =$ the square of the radius of gyration about its center, $\mu=$ the coefficient of friction, $s=$ the distance passed over by the center in the time t from the beginning of motion, perfect rolling being assumed, $R=$ the normal reaction of the plane, $F=$ the friction, $\phi=$ the angular rotation of the sphere, $g=$ the acceleration of gravity, and $m=$ the mass of the sphere.

Resolving parallel and perpendicular to the plane, and taking moments about the center of the sphere, $m\dfrac{d^2s}{dt^2} = mg \sin\theta - F \ldots (1)$,

$$R = mg \cos\theta \ldots (2) \text{ and } mk^2\dfrac{d^2\phi}{dt^2} = Fa \ldots (3).$$

We have, also, $s = a\phi \ldots (4)$, and then $\dfrac{ds}{dt} = a\dfrac{d\phi}{dt} \ldots (5)$,

$$\dfrac{d^2s}{dt^2} = a\dfrac{d^2\phi}{dt^2} \ldots (6).$$

Eliminating F from (1) and (3) by aid of (6) we find

$$\dfrac{d^2s}{dt^2} = \dfrac{a^2g \sin\theta}{a^2+k^2} \ldots (7), \text{ and then } \dfrac{d^2\phi}{dt^2} = \dfrac{ag \sin\theta}{a^2+k^2} \ldots (8).$$

Substituting in (3) we have $F = \dfrac{mgk^2 \sin\theta}{a^2+k^2} \ldots (9)$. Then $\mu = \dfrac{F}{R}$

$= \dfrac{k^2}{a^2+k^2} \tan\theta \ldots (10)$.

Substituting $\mu R = \mu\, mg \cos\theta \ldots (11)$ for F in (1) and (2), and integrating once, $\dfrac{ds}{dt} = gt(\sin\theta - \mu \cos\theta) + c \ldots (12)$. $\dfrac{d\phi}{dt} = \dfrac{\mu agt}{k^2} \cos\theta + \phi' \ldots (13)$

ϕ' being the initial angular velocity.

From (12) and (13) we have

$$\frac{ds}{dt} - a\frac{d\phi}{dt} = gt\cos\theta\frac{a^2+k^2}{k^2}\left(\frac{k^2}{a^2+k^2}\tan\theta - \mu\right)\dots(14).$$

By hypothesis, $\mu < \frac{k^2}{a^2+k^2}\tan\theta$ or $<\frac{2}{7}\tan\theta$, and therefore $\frac{ds}{dt} - a\frac{d\phi}{dt}$ is positive, showing that all the motion is not rolling.

III. **Solution by Professor P. H. PHILBRICK, M. S., C. E., Lake Charles, Louisiana.**

Let $a=$the radius of the sphere, O the origin, A the point of the sphere in contact with the plane at O at starting and P the point in contact at the end of the time t. $OP=x$.

Let $ACP=\phi$, $R=$pressure on the plane, $F=$the sliding friction, and $\mu=$coefficient of sliding friction$=\frac{F}{R}$.

Suppose the plane just rough enough to prevent sliding.

Then the equation for translation is $m\frac{d^2x}{dt^2} = mg\sin\theta - F\dots(1)$,

and for rotation $mk_1^2\frac{d^2\phi}{dt^2} = aF\dots(2)$, in which $k_1=$the radius of gyration and $\therefore k_1^2=\frac{2}{5}a^2$.

Also $x=OP=$arc $AP=a\phi$; and differentiating twice gives, $\frac{d^2x}{dt^2}=a\frac{d^2\phi}{dt^2}\dots(3)$.

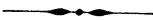Multiply (1) by a and add to (2) and get, $ma\frac{d^2x}{dt^2}+mk_1^2\frac{d^2\phi}{dt^2} = mg\sin\theta\dots(4)$.

Substituting from (3) we get, $\frac{d^2\phi}{dt^2} = \frac{a}{a^2+k_1^2}g\sin\theta$.

From (2) and (5), $F=\frac{mk_1^2}{a}\cdot\frac{d^2\phi}{dt^2} = \frac{mk_1^2}{a}\cdot\frac{a}{a^2+k_1^2}g\sin\theta=\frac{mk_1^2}{a^2+k_1^2}g\sin\theta$.

Also $R=mg\cos\theta$. $\therefore \mu=\frac{F}{R} = \frac{k_1^2}{k_1^2+a^2}\tan\theta=\frac{2}{7}\tan\theta$.

Hence if μ is less than $\frac{2}{7}\tan\theta$, it will all be utilized in causing rolling but it is insufficient to prevent some sliding.

DIOPHANTINE ANALYSIS.

SOLUTIONS TO PROBLEMS.

11. Proposed by **ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

Find three whole numbers such that the square of the sum of any two of them diminished by the square of the other number shall be a square.